 national accelerator laboratory	Author S. C. Snowdon	Section Theory	Page 1 of 11
	Date September 16, 1968	Category 0106	Serial TM-44

Subject

CONVENTIONAL BENDING MAGNETS FOR STORAGE RING

Aperture Considerations

Keil¹ has calculated the aperture requirements for injecting a 100 GeV beam from the main accelerator into a storage ring having one-third the radius of the main ring. In order to reduce the radial aperture, he employs a phase plane interchange during transfer. A. van Steenberg² and Keil³ suggested that a full aperture kicker be employed to eliminate the screening distance between the injected beam and the stacked beam.

The option developed here utilizes the phase plane interchange but not the full aperture kicker, other options will be considered later. Keil's calculations are developed further with the aid of a simple FODO lattice whose basic parameters were given by Teng. We further adopt the beam emittances listed in the Design Report at 200 GeV and alter them by the momentum ratio for 100 GeV.

Beam Emittance in Main Accelerator

$$E_H = .23 \times \frac{200.94}{100.93} \pi = .458 \pi \text{ mm-mrad}$$

$$E_V = .09 \times \frac{200.94}{100.93} \pi = .179 \pi \text{ mm-mrad}$$

Interchange of Phase Space in Transport Channel

Assume a dilution factor of 2 in transfer.

$$E_H = 2 \times .179 \pi = .358 \pi \text{ mm-mrad},$$

$$E_V = 2 \times .458 \pi = .916 \pi \text{ mm-mrad}.$$

Three-Turn Radial Injection into Storage Ring

$$E_H = (2 + 1/3)^2 \times .358 \pi = 1.949 \pi \text{ mm-mrad},$$

$$E_V = .916 \pi \text{ mm-mrad}.$$

Beam Radii

Divide the magnet between the focusing and defocusing quadrupoles into four magnets to permit reduction of the effects of sagitta and more frequent access for pumps. From the focusing quadrupole (QF) designate the magnets B11, B12, B21, B22, QD. Using $a_x = (\beta_x E_H / \pi)^{1/2}$, $a_y = (\beta_y E_V / \pi)^{1/2}$, the program TRACEX gives

	a_x (mm)	a_y (mm)
QF	8.45	2.77
B11	7.31	3.71
B12	6.50	4.25
B21	5.69	4.79
B22	4.99	5.37
QD	4.04	5.79

Closed-Orbit Errors

Assuming rms errors of

$$\Delta B/B = \Delta L_{\text{MAG}}/L_{\text{MAG}} = \Delta \theta_{\text{MAG}} = 5 \times 10^{-4}$$

$$\Delta X(\text{QUAD}) = \Delta Y(\text{QUAD}) = 10^{-4} \text{ m.}$$

$$\begin{aligned} \langle V_x \rangle &= \frac{2 \times 72}{4 \times 1/2} \cdot \left(\frac{7.35 \times 20}{4 \times 3366.8} \right)^2 \cdot (27.45 + 21.67 + 16.59 + 12.76) \\ &\times (25 \times 10^{-8} + 25 \times 10^{-8}) + \frac{72}{4 \times 1/2} \cdot \left(\frac{384 \times 10^{-4}}{3366.8} \right)^2 \cdot (36.64 + 8.38) \\ &= 54.74 \times 10^{-8} \text{ m.} \end{aligned}$$

$$\begin{aligned} \langle V_y \rangle &= \frac{2 \times 72}{4 \times 1/2} \cdot \left(\frac{7.35 \times 20}{4 \times 3366.8} \right)^2 \cdot (14.99 + 19.67 + 25.09 + 31.45) \\ &\times (25 \times 10^{-8} + 25 \times 10^{-8}) + \frac{72}{4 \times 1/2} \cdot \left(\frac{384 \times 10^{-4}}{3366.8} \right)^2 \cdot (8.38 + 36.64) \\ &= 60.20 \times 10^{-8} \text{ m.} \end{aligned}$$

Keil⁴ and Laslett⁵ indicate that, with 98 percent probability, the closed-orbit deviation is less than $(3.5/\sqrt{2}) \cdot \sqrt{\beta \langle V \rangle}$. L. Smith⁶ indicates that with 70 percent probability the closed-orbit deviation is less than $(2.5/\sqrt{2}) \cdot \sqrt{\beta \langle V \rangle}$. The 70 percent allowances are listed below

	CO _x (mm)	CO _y (mm)
QF	± 7.93	± 3.96
B11	± 6.87	± 5.31
B12	± 6.09	± 6.09
B21	± 5.33	± 6.89
B22	± 4.67	± 7.70
QD	± 3.79	± 8.32

Sagitta (8 × 72 Magnet Blocks)

$$\frac{(7.35/4)^2}{8 \times 168.34} = 2.507 \text{ mm} = \pm 1.25 \text{ mm.}$$

Gradient Errors ($\delta B' = .01 B'$)

$$\langle \delta v^2 \rangle = \frac{72}{4\pi^2} \cdot \left(\frac{.01 \times 384}{3366.8} \right)^2 \cdot \left[(36.64)^2 + (8.38)^2 \right]$$

$$\delta v = .0579; \Delta B/B = \pi \times .0579/1.0 = .1819.$$

This gives

	δx (mm)	δy (mm)
QF	$\pm .74$	$\pm .24$
B11	$\pm .64$	$\pm .32$
B12	$\pm .57$	$\pm .37$
B21	$\pm .50$	$\pm .42$
B22	$\pm .43$	$\pm .47$
QD	$\pm .35$	$\pm .50$

Radial Spread in Beam Due to Momentum Spread

	Injected	Stacked
$\Delta p/p$	$\pm 10^{-3}/20$	$\pm 10^{-3}$
QF	$\pm .08$	± 1.62
B11	$\pm .08$	± 1.51
B12	$\pm .07$	± 1.35
B21	$\pm .06$	± 1.16
B22	$\pm .05$	$\pm .98$
QD	$\pm .04$	$\pm .84$

Beam Diameters

	Injected		Stacked	
	Horiz. (mm)	Vert. (mm)	Horiz. (mm)	Vert. (mm)
QF	17.1	5.5	20.1	5.5
B11	14.8	7.4	17.6	7.4
B12	13.1	8.5	15.7	8.5
B21	11.5	9.6	13.7	9.6
B22	11.1	10.7	11.9	10.7
QD	8.2	11.6	9.8	11.6

Vertical Space Required for Vacuum Chamber and Insulation

6 mm total

Space Required Between Injected and Stacked Beams

QF	16.0 mm
B11	15.1
B12	13.5
B21	11.6
B22	9.8
QD	8.4

The width of the good-field region is obtained by adding the horizontal injected beam diameter, the space required between injected and stacked beams, the horizontal stacked beam diameter and the space allotted for beam displacement and growth. Single particle detuning by the required 10^{15} particles determines the average vertical aperture, 1.125 in. (B_1) and 1.500 in. (B_2) being chosen to tailor the apertures to the beam size in the circumstance that the beam fills the aperture.

Required Magnet Apertures

	Good Field Width (in.)	Vert. Gap (in.)
QF	2.875	1.125
B11	2.561	1.125
B12	2.290	1.125
B21	2.006	1.500
B22	1.406	1.500
QD	1.461	1.500

For construction choose

B_1	2.625	1.125
B_2	2.000	1.500

Space Charge Limit ($\beta_{AV} = 19.56 \text{ m}$, $X_p(AV) = 1.22 \text{ m}$)

$$a = \sqrt{1.949 \times 19.56} + 1.22 = 7.39 \text{ mm}$$

$$b = \sqrt{.916 \times 19.56} = 4.23 \text{ mm}$$

$$\left\langle 1/H^2 \right\rangle = 1/2 \left(\frac{1}{(2.2575)^2} + \frac{1}{(3.21)^2} \right); \quad H = 2.61 \text{ cm}$$

$$\left\langle 1/G^2 \right\rangle = \frac{1}{2 \times 2.0} \left(\frac{1}{(2.8575)^2} + \frac{1}{(3.81)^2} \right); \quad G = 4.57 \text{ cm}$$

$$N (\text{Incoh.} - \text{Transv.}) = 1.6 \times 10^{15} \text{ particles.}$$

Magnet Design

For a rough design one may utilize the properties of a semi-infinite condenser. A uniform vertical field above the equipotential surface, $V = 1/2 V_0$, where V_0 is potential of semi-infinite plate matches the field distribution below this equipotential surface. More suitable termination following the method used for the booster magnet design will permit a narrower pole width for the same gradient tolerance. Experience with the booster magnet design is used to modify the pole widths calculated from the semi-infinite condenser approach.

Allowed Quadrupole and Sextupole Tolerances

A gradient Hy' in the bending magnets introduces a betatron tune shift of

$$\Delta\nu = \frac{2 \times 72 \times 1.8375}{4\pi \times 168.34 \times 20} Hy' (31.45 + 25.09 + 19.67 + 14.99) = 0.57 Hy'.$$

For $\Delta v = 0.1$,

$$H_y' = .175 \text{ kG/m.}$$

At the location for which the semi-infinite condenser problem yields this gradient, one finds

$$H_y'' = 19 \text{ kG/m } (B_1); 14 \text{ kG/m } (B_2)$$

These may be adopted as the maximum quadrupole and sextupole effects introduced by the bending magnet. Better pole designs will permit one to achieve these values with narrower poles.

Pole Widths

Using the semi-empirical procedures indicated, one determines that the pole widths at the base of the pole may be taken as

$$W = 6.00 \text{ in. } (B_1); 6.50 \text{ in. } (B_2).$$

Results

Table I includes the magnet design data for the case described here. Simple magnetic circuit calculations using C1010 steel have been used to estimate the required excitations. Note that one coil surrounds two magnets. The coil design is considered to be reasonable but not necessarily optimum. A current density of about 3500 A/in.^2 has been used as a guide in determining reasonable power levels. Figures 1 and 2 show the magnet and coil cross sections for the cases described.

REFERENCES

- ¹E. Keil, Aperture Requirements at Injection into NAL Storage Ring,
National Accelerator Laboratory Internal Report FN-169 (Aug. 27, 1968).
- ²A. van Steenbergen, NAL Storage Ring Study, Aug. 1968.
- ³E. Keil, NAL Storage Ring Study, Aug. 1968.
- ⁴E. Keil, CERN Report AR/Int. SG/65-3 (Feb. 10, 1965).
- ⁵L. J. Laslett, Lawrence Radiation Laboratory Internal Report
UCID-10158 (May, 1965).
- ⁶Lloyd Smith, private communication.

Table I. Storage Ring Magnet Parameters

	<u>B₁₁</u>	<u>B₁₂</u>	<u>B₂₁</u>	<u>B₂₂</u>
Magnet Length (m)	1.8375	1.8375	1.8375	1.8375
B ₀ at Magnet Center (kG)	20.0	20.0	20.0	20.0
B' ₀ at Good Field Edge (kG/m)	0.175	0.175	0.175	0.175
B'' at Good Field Edge (kG/m ²)	19.0	19.0	14.0	14.0
Magnet Width (in.)	22.50	22.50	26.75	26.75
Magnet Height (in.)	13.75	13.75	14.75	14.75
Pole Width (in.)	6.00	6.00	6.50	6.50
Magnet Gap (in.)	1.125	1.125	1.50	1.50
Good Field Width (in.)	2.625	2.625	2.00	2.00
Coil Window Width (in.)	4.125	4.125	5.50	5.50
Coil Window Height (in.)	5.50	5.50	5.50	5.50
Coil Turns	48		64	
Conductor Width (in.)	.625		.625	
Conductor Height (in.)	.625		.625	
Conductor Hole Dia. (in.)	.375		.375	
Conductor Corner Radius (in.)	.0625		.0625	
Spacing Between Cond. (in.)	.053		.053	
Conductor Current (A)	1058		1058	
Resistance (ohms)	.0423		.0573	
Power (kW)	47.3		64.2	
Inductance (H)	.0544		.0819	
Stored Energy (kJ)	30.4		45.8	
Cooling Water Press. (psi)	200.0		200.0	
No. Water Paths/Coil	4.0		4.0	
Water Temp. Rise (°C)	14.0		22.0	
Magnet Iron Weight (lbs)	5211	5211	6569	6569
Copper Coil Weight (lbs)	1425		1933	
Stacked Beam Sp. Ch. Lim. (10 ¹⁵)			1.6	
Total Mag. Wt. (2 Rings - Tons)			3392	
Total Copper Wt. (2 Rings - Tons)			484	
Total Power (2 Rings - MW)			32.1	
Total Stored Energy (2 Rings - MJ)			21.9	
Total Water Flow (2 Rings - GPM)			6929	

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NATIONAL ACCELERATOR LABORATORY INTERSECTING STORAGE RING CROSS-SECTION MONITOR MAGNET, IS	
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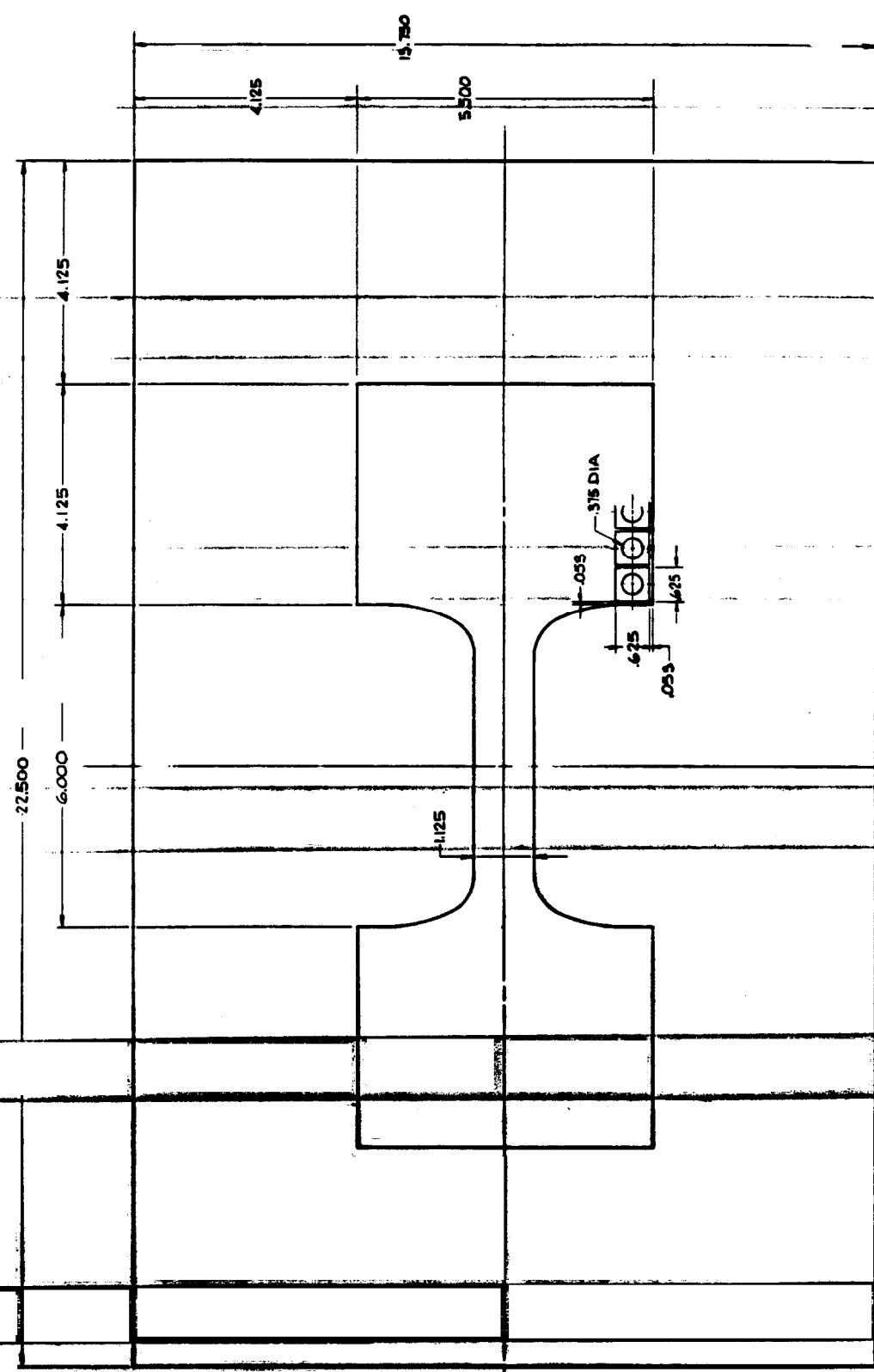


Fig. 1

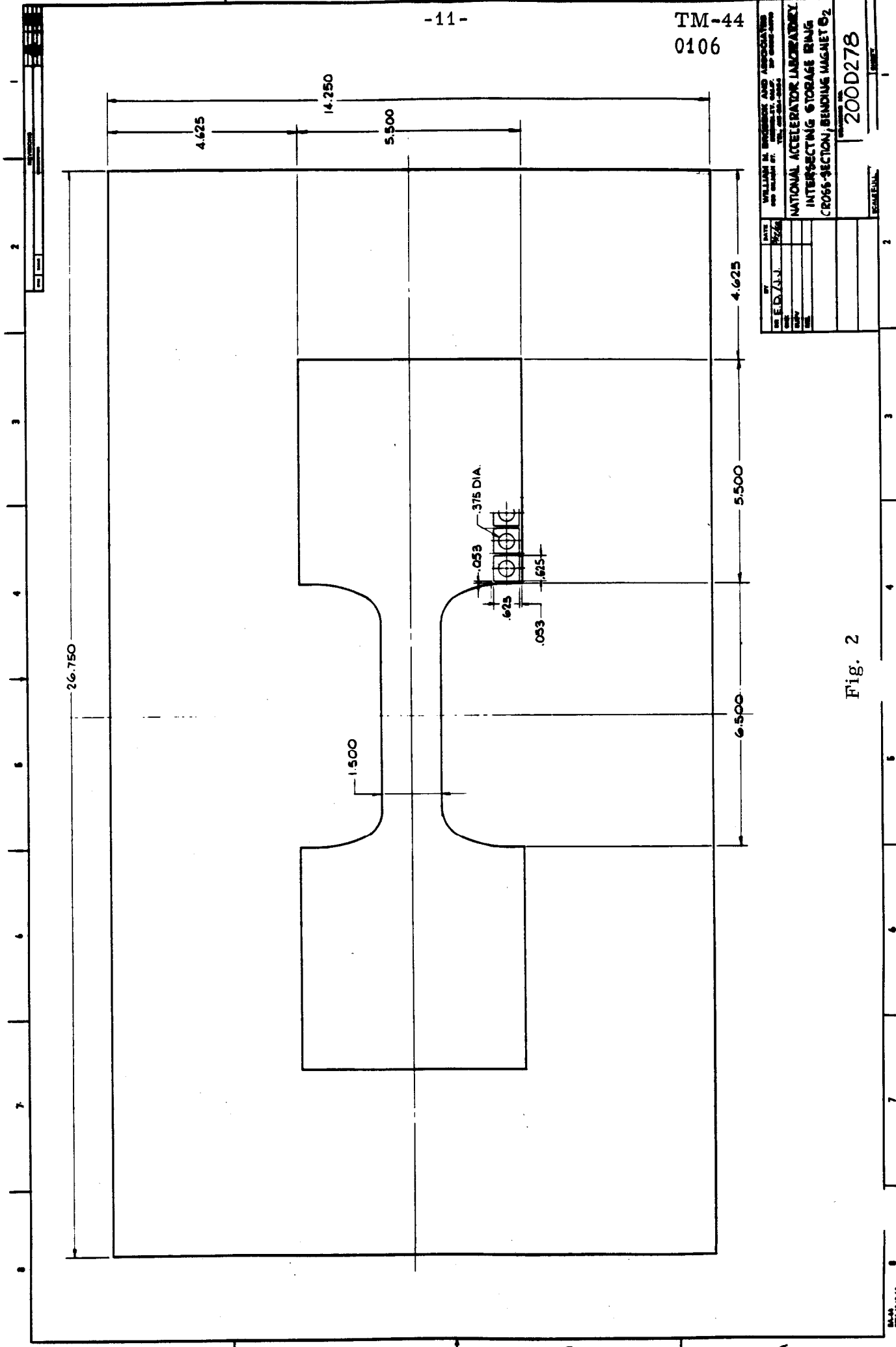


Fig. 2

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